Decision Fusion With Unknown Sensor Detection Probability

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***Abstract*—In this letter we study the problem of channel-aware decision fusion when the sensor detection probability is not known at the decision fusion center. Several alternatives proposed in the literature are compared and new fusion rules (namely "ideal sensors" and "locally-optimum detection") are proposed, showing attractive performance and linear complexity. Simulations are provided to compare the performance of the aforementioned rules.**

***Index Terms*—Decentralized detection, decision fusion, locally-optimum detection (LOD), wireless sensor networks (WSNs).**

I. Introduction

THERE is a vast literature on decision fusion (DF) in wireless sensor networks (WSNs) [1]. There are cases where the uniformly more powerful test is independent from the sensors performance [2], however most authors assume that the sensor performance are known to the DF center (DFC) [3], [4], [5] to derive the likelihood ratio test (LRT). Two approaches are used to relax this requirement: (*i*) the use of sub-optimal rules (e.g. the *diversity* statistics in [3], [4], [6], (*ii*) assuming the probability of false alarm at the sensors is known and estimating the detection probability as part of a composite hypothesis test [7].

In this letter we study channel-aware DF when the *false-alarm* probability of the generic sensor is *known*, while the *detection* probability is *unknown*. First, we perform a detailed comparison of existing fusion alternatives, not *requiring* knowledge of *sensor detection probability*, based on the approaches (*i*) (i.e. the *counting rule* [1]) and (*ii*) (i.e. the rule proposed in [7], denoted here as "*Wu rule*"). The comparison is strengthened by a theoretical analysis in the case of a large number of sensors, based on deflection measures [8]. Also, we derive two novel rules, based on "ideal sensors" assumption (approach (*i*)) [3], [4], [9] and locally-optimum detection (approach (*ii*)) [10]. For all the considered rules high/low signal-to-noise ratio (SNR) optimality properties are established in a scenario with identical sensors and a discussion on complexity and required system knowledge is reported. Finally, the case of non-identical sensors is considered.

The paper is organized as follows: Section II introduces the model; in Section III we derive and study the fusion rules, while in Section IV we generalize the analysis to the case of non-identical sensors; in Section V we compare the presented rules and confirm the theoretical findings through simulations; finally in Section IV we draw some conclusions.

II. System Model

The model is described as follows[1]. We consider a decentralized binary hypothesis test, where K sensors are used to discriminate between the hypotheses of the set $\mathcal{H} = \{\mathcal{H}_0, \mathcal{H}_1\}$, representing the absence ($\mathcal{H}_0$) or the presence ($\mathcal{H}_1$) of a phenomenon of interest. The a priori probability of $\mathcal{H}_i \in \mathcal{H}$ is denoted $P(\mathcal{H}_i)$. The kth sensor, $k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$, takes a binary decision $d_k \in \mathcal{H}$ about the phenomenon on the basis of its own measurements, which is then mapped to a symbol $b_k \in \{0, 1\}$; without loss of generality (w.l.o.g.) we assume that $d_k = \mathcal{H}_i$ maps into $b_k = i$, $i \in \{0, 1\}$.

The quality of the kth sensor decisions is characterized by the conditional probabilities $P(b_k|\mathcal{H}_j)$: we denote $P_D \triangleq P(b_k = 1|\mathcal{H}_1)$ and $P_F \triangleq P(b_k = 1|\mathcal{H}_0)$ the probabilities of detection and false alarm of the kth sensor, respectively. Initially, we assume conditionally independent and identically distributed (i.i.d.) decisions; this restriction will be relaxed in Section IV. Also we assume $P_D > P_F$, because of the informativeness of the decision at each sensor. Differently from [4], we assume that P_F *is known* at the DFC, but on the other hand that the true P_D *is unknown*, as studied in [7].

The kth sensor communicates to the DFC over a dedicated binary symmetric channel (BSC) and the DFC observes a noisy binary-valued signal y_k, that is $y_k = b_k$ with probability $(1 - P_{e,k})$ and $y_k = (1 - b_k)$ with probability $P_{e,k}$, which we collect as $\boldsymbol{y} \triangleq [y_1 \quad \cdots \quad y_K]^t$. We denote $P_{e,k}$ the bit-error probability (BEP) of the kth link[2]. The BSC model arises when separation between sensing and communication layers is performed in the design (a "*decode-then-fuse*" approach [6]).

[1]*Notation* - Lower-case bold letters denote vectors, with a_n being the nth element of $\boldsymbol{a}$; $\|\boldsymbol{a}\|_p$ denotes the ℓ_p-norm of $\boldsymbol{a}$; upper-case calligraphic letters, e.g. $\mathcal{A}$, denote finite sets; $\mathbb{E}\{\cdot\}$, $\mathrm{var}\{\cdot\}$ and $(\cdot)^t$ denote expectation, variance and transpose, respectively; $P(\cdot)$ and $p(\cdot)$ are used to denote probability mass functions (pmf) and probability density functions (pdf), respectively, while $P(\cdot|\cdot)$ and $p(\cdot|\cdot)$ their corresponding conditional counterparts; $\mathcal{N}_{\mathbb{C}}(\mu, \sigma^2)$ denotes a proper complex-valued Gaussian pdf with mean μ and variance σ^2, while $\mathcal{Q}(\cdot)$ is the complementary cumulative distribution function of a standard normal random variable; $\mathcal{U}(a, b)$ denotes a uniform pdf with support $[a, b]$; finally the symbol $\sim$ means "distributed as".

[2]Throughout this letter we make the reasonable assumption $P_{e,k} \leq \frac{1}{2}$.

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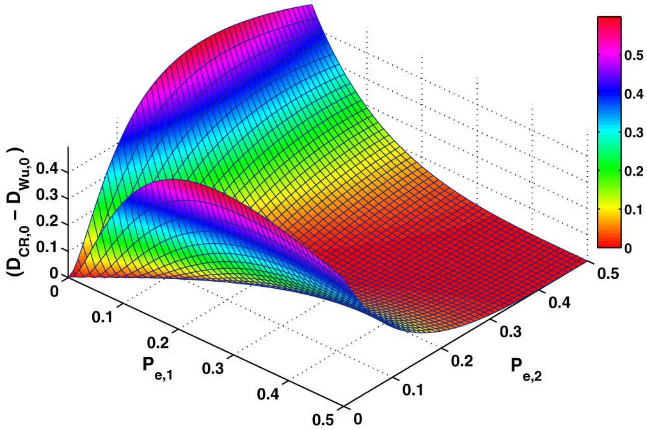

Fig. 1. $(D_{\mathrm{CR},0} - D_{\mathrm{Wu},0})$ for $K = 2$ sensors as a function of $\{P_{e,1}, P_{e,2}\}$, conditionally i.i.d. decisions $(P_F, P_D) = (0.05, 0.5)$.

The pmf of $\boldsymbol{y}$ is the same under both $\mathcal{H}_0$ and $\mathcal{H}_1$, except for the value of the unknown parameter $P_1 \triangleq P(b_k = 1|\mathcal{H})$. Denoting the pmf with $P(\boldsymbol{y}; P_1)$, the test is summarized as:

$$\mathcal{H}_0 : P_1 = P_F; \qquad \mathcal{H}_1 : P_1 > P_F; \tag{1}$$

which is recognized as a *one-sided* (composite) test [11].

III. Fusion Rules

The final decision at the DFC is performed as a test comparing a signal-dependent fusion rule $\Lambda(\boldsymbol{y})$ and a fixed threshold γ:

$$\begin{array}{c} \hat{\mathcal{H}} = \mathcal{H}_1 \\ \Lambda(\boldsymbol{y}) > rless\ \gamma \\ \hat{\mathcal{H}} = \mathcal{H}_0 \end{array} \tag{2}$$

where $\hat{\mathcal{H}}$ denotes the estimated hypothesis. Hereinafter we propose different fusion rules for the considered problem.

(Clairvoyant) LRT - in this case we assume that also P_D is known at the DFC. The explicit expression of the LRT is given by

$$\begin{aligned} \Lambda_{\mathrm{LRT}} &\triangleq \ln\left[\frac{P(\boldsymbol{y}; P_1 = P_D)}{P(\boldsymbol{y}; P_1 = P_F)}\right] = \sum_{k=1}^{K} \ln\left[\frac{P(y_k; P_1 = P_D)}{P(y_k; P_1 = P_F)}\right] \\ &= \sum_{k=1}^{K} \left\{ y_k \ln\left[\frac{\alpha_k(P_D)}{\alpha_k(P_F)}\right] + (1 - y_k)\ln\left[\frac{\beta_k(P_D)}{\beta_k(P_F)}\right]\right\} \end{aligned} \tag{3}$$

where $\alpha_k(P_1) \triangleq P(y_k = 1; P_1) = ((1 - 2P_{e,k}) \cdot P_1 + P_{e,k})$ and $\beta_k(P_1) \triangleq P(y_k = 0; P_1) = (1 - \alpha_k(P_1))$. It is apparent that Eq. (3) should not be intended as a realistic element of comparison, but rather as an optimistic upper bound on the achievable performance (since it makes use of both P_D and P_F). Differently, in this letter it is assumed that $P_{e,k}$ can be easily obtained, as in [12].

Ideal sensors (IS) rule - we obtain this rule by assuming that the sensing phase works ideally, that is $(P_D, P_F) = (1, 0)$. This simplifying assumption is exploited in Eq. (3), thus leading to:

$$\Lambda_{\mathrm{IS}} \triangleq \sum_{k=1}^{K} (2\, y_k - 1) \ln\left[\frac{1 - P_{e,k}}{P_{e,k}}\right]. \tag{4}$$

TABLE I
Comparison of Rules w.r.t. System Knowledge Requirements

Fusion rule	Required parameters
(Clairvoyant) LRT	P_D, P_F, $P_{e,k}$
LOD rule	P_F, $P_{e,k}$
IS rule	$P_{e,k}$
CR	none
Wu rule [7]	P_F, $P_{e,k}$

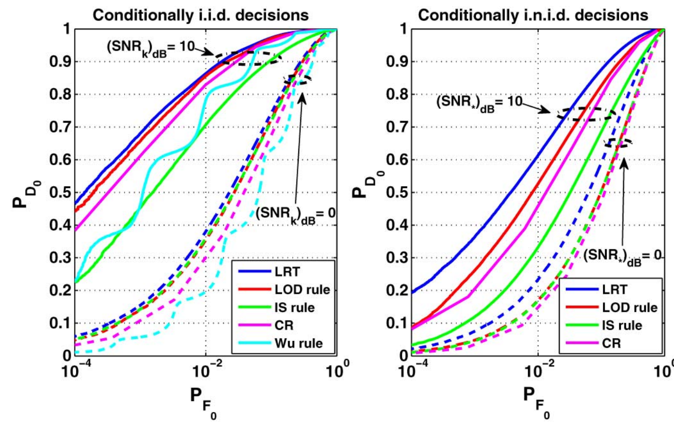

Fig. 2. P_{D_0} vs. P_{F_0}; WSN with $K = 10$ and $(\mathrm{SNR}_k)_{\mathrm{dB}} \in \{0, 10\}$ (resp. $(\mathrm{SNR}_\star)_{\mathrm{dB}} \in \{0, 10\}$); $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ (resp. $(P_{FU}, P_{DE}) = (0.2, 0.6)$) for conditionally i.i.d. (resp. i.n.i.d.) decisions.

The assumption behind Eq. (4) is not new: indeed it was considered in [3], [4], [9] to derive sub-optimal rules (i.e. the *maximum ratio* and the *equal gain combiners*) under different communication models.

Locally-optimum detection (LOD) rule - the one-sided nature of the test considered allows to pursue a LOD-based approach, whose implicit expression is given by [10], [11]

$$\Lambda_{\mathrm{LOD}} \triangleq \left.\frac{\partial \ln[P(\boldsymbol{y}; P_1)]}{\partial P_1}\right|_{P_1 = P_F} \times \left(\sqrt{I(P_F)}\right)^{-1}, \tag{5}$$

where $I(P_1)$ represents the *Fisher information* (FI), that is:

$$I(P_1) \triangleq \mathbb{E}\left\{\left(\frac{\partial \ln[P(\boldsymbol{y}; P_1)]}{\partial P_1}\right)^2\right\}. \tag{6}$$

The explicit form of Λ_{LOD} is shown in Eq. (7) at the top of the next page; the derivation is given as supplementary material.

$$\begin{aligned} \Lambda_{\mathrm{LOD}} = &\left(\sum_{k=1}^{K} \frac{(1 - 2\,P_{e,k}) \cdot [(y_k - P_{e,k}) - (1 - 2\,P_{e,k})P_F]}{\alpha_k(P_F)\beta_k(P_F)}\right) \\ &\times \left(\sqrt{\sum_{k=1}^{K} \frac{(1 - 2\,P_{e,k})^2}{\alpha_k(P_F)\beta_k(P_F)}}\right)^{-1} \end{aligned} \tag{7}$$

Counting rule (CR) - this rule is widely used in DF (due to its simplicity and no requirements on system knowledge) and it is obtained by assuming that the communication channels are ideal, i.e.

$$\Lambda_{\mathrm{CR}} \triangleq \sum_{k=1}^{K} y_k, \tag{8}$$

since $P_{e,k} = 0$ entails $\alpha_k(P_1) = P_1$ and irrelevant terms are incorporated in γ through Eq. (2).

Wu rule [7] - this rule was proposed by *Wu et al.* and it was shown to outperform a GLRT rule for all the scenarios considered. We report only the final result and omit the details. First an *approximate*[3] *maximum-likelihood* (ML) estimate of P_D is obtained as

$$\hat{P}_D \triangleq \frac{1}{K}\sum_{k=1}^{K}\left[(1+2\,P_{e,k})y_k - P_{e,k}\right], \tag{9}$$

then the following statistic is employed:

$$\Lambda_{\mathrm{Wu}} \triangleq (\hat{P}_D - P_F). \tag{10}$$

Remark: when $P_{e,k} = P_e$ all the rules are *equivalent*[4]. Thus, when the SNR goes to infinity (i.e. $P_{e,k} \to 0$) all the rules undergo the same performance. The only exception is Λ_{IS}, since $\lim_{P_{e,k}\to 0}\Lambda_{\mathrm{IS}} = +\infty$ (such a difference leads to a loss in performance, as shown in Section V). Differently, in the low SNR regime their behaviour is significantly different, as shown by the following proposition.

Proposition 1: When the SNR is low at each link, Λ_{IS} *and* Λ_{LOD} *approach* Λ_{LRT}*, while* Λ_{Wu} *does not.*

Proof: Given as supplementary material. ■

It is worth noting that: (*i*) Prop. 1 does not require $P_{e,k}$ to be equal and that (*ii*) the low-SNR optimality of Λ_{IS} in Prop. 1 is coherent with the results shown in [4], [5], [6].

Wu rule vs CR deflection comparison: since all the considered rules are equivalent to scaled sums of independent Bernoulli random variables, the pmf $P(\Lambda|\mathcal{H}_i)$ is intractable [7]. Hence we rely on the so-called *deflection measures* [8] $D_i \triangleq \frac{(\mathbb{E}\{\Lambda|\mathcal{H}_1\}-\mathbb{E}\{\Lambda|\mathcal{H}_0\})^2}{\mathrm{var}\{\Lambda|\mathcal{H}_i\}}$ to perform a theoretical comparison between Λ_{CR} and Λ_{Wu}. This choice is justified since, as K grows large, $P(\Lambda|\mathcal{H}_i)$ converges to a Gaussian pdf (in virtue of the *central limit theorem* [13]). It can be shown that for CR and Wu rule the deflections assume the following expressions:

$$D_{\mathrm{CR},i} = \frac{\left(\sum_{k=1}^{K} m_k\right)^2}{\sum_{k=1}^{K} c_{i,k}}, \quad D_{\mathrm{Wu},i} = \frac{\left(\sum_{k=1}^{K} n_k m_k\right)^2}{\sum_{k=1}^{K} n_k^2 c_{i,k}}, \tag{11}$$

where $m_k \triangleq (1-2P_{e,k})(P_D - P_F)$, $n_k \triangleq (1+2P_{e,k})$, $c_{0,k} \triangleq \alpha_k(P_F)(1-\alpha_k(P_F))$ and $c_{1,k} \triangleq \alpha_k(P_D)(1-\alpha_k(P_D))$. W.l.o.g., we assume $P_{e,k} \geq P_{e,k+1}$, which in turn gives $m_k \leq m_{k+1}$, $n_k \geq n_{k+1}$ and $c_{i,k} \geq c_{i,k+1}$ (since we assume $P_{e,k} \leq \frac{1}{2}$). Consequently, the *Chebyshev's sum inequalities* [14] $\sum_{k=1}^{K} n_k m_k \leq \frac{1}{K}(\sum_{k=1}^{K} m_k)(\sum_{k=1}^{K} n_k)$ and $\sum_{k=1}^{K} n_k^2 c_{i,k} \geq \frac{1}{K}(\sum_{k=1}^{K} c_{i,k})(\sum_{k=1}^{K} n_k^2)$ hold, which jointly give:

$$D_{\mathrm{Wu},i} \leq D_{\mathrm{Wu},i}\left(\frac{\sqrt{K}\|\boldsymbol{n}\|_2}{\|\boldsymbol{n}\|_1}\right)^2 \leq D_{\mathrm{CR},i} \tag{12}$$

where $\boldsymbol{n} \triangleq [\,n_1 \quad \cdots \quad n_K\,]^t$ and the first inequality arises from the application of *Cauchy-Schwartz inequality* [15] to $\|\boldsymbol{n}\|_1$.

In Fig. 1 we illustrate $(D_{\mathrm{CR},0} - D_{\mathrm{Wu},0})$ (in a WSN with $K = 2$) as a function of $(P_{e,1}, P_{e,2})$ in a scenario with $(P_F, P_D) = (0.05, 0.5)$. It is confirmed that $D_{\mathrm{Wu},i}$ is always dominated by $D_{\mathrm{CR},i}$ and that the effect is more pronounced when $P_{e,1}$ and $P_{e,2}$ differ significantly (indeed when $P_{e,1} = P_{e,2}$, Λ_{Wu} is equivalent to Λ_{CR}). The superiority of Λ_{CR} is also confirmed via the results in Section V.

Discussion on complexity and system knowledge: as discussed in [7], Λ_{Wu} being *affine* in $\boldsymbol{y}$ (cf. Eqs. (9)-(10)) is one of the main advantages w.r.t. the GLRT. This feature reduces the complexity at the DFC and facilitate performance analysis. Since all the considered alternatives (i.e. Λ_{IS}, Λ_{LOD} and Λ_{CR}) are also affine functions of $\boldsymbol{y}$, *they exhibit the same advantages*. On the other hand, as summarized in Table I, the presented fusion rules have different requirements in terms of system knowledge. In fact, while Λ_{LOD} and Λ_{Wu} entail the same requirements (i.e. P_F and $P_{e,k}$)), Λ_{IS} only needs $P_{e,k}$. Finally, Λ_{CR} does not require any parameter for its implementation.

IV. Extension to Non-Identical Sensors Scenario

In this section we generalize the proposed rules to a scenario with non-identical sensors, i.e. $(P_{D,k}, P_{F,k})$, $k \in \mathcal{K}$, where $P_{F,k}$ is *known* but $P_{D,k}$ is *still unknown* at the DFC.

(Clairvoyant) LRT - Λ_{LRT} is obtained by replacing $\alpha_k(P_D)$ (resp. $\alpha_k(P_F)$) with $\alpha_k(P_{D,k})$ (resp. $\alpha_k(P_{F,k})$) in Eq. (3).

LOD rule - the rule is extended to conditionally independent and non-identically distributed (i.n.i.d.) decisions as:

$$\breve{\Lambda}_{\mathrm{LOD}} \triangleq \sum_{k=1}^{K} \left.\frac{\partial \ln\left[P(y_k; P_1)\right]}{\partial P_1}\right|_{P_1 = P_{F,k}} \times \sqrt{\mathrm{I}_k(P_{F,k})}^{-1} \tag{13}$$

CR, IS and Wu fusion rules - in this scenario Λ_{IS} retains the same form as in Eq. (4), while it is apparent that $\Lambda_{\mathrm{CR}} = \sum_{k=1}^{K} y_k$ does not arise from the assumption $P_{e,k} = 0$ in Λ_{LRT}. Nonetheless we will still keep Λ_{CR} in the comparison of Section V, since it represents a natural "$P_{D,k}$-unaware" alternative. Finally, we discard Eq. (10) from our comparison, since the (approximate) ML estimate in Eq. (9) is performed assuming $P_{D,k} = P_D$.

V. Numerical Results

We compare the performance of the proposed rules in terms of system false alarm and detection probabilities, defined as

$$P_{F_0} \triangleq \Pr\{\Lambda > \gamma|\mathcal{H}_0\}, \qquad P_{D_0} \triangleq \Pr\{\Lambda > \gamma|\mathcal{H}_1\}, \tag{14}$$

respectively, where Λ is the generic rule employed at the DFC.

Similarly as in [7], we consider communication over a Rayleigh fading channel via on-off keying, i.e. $x_k = h_k b_k + w_k$, where $x_k \in \mathbb{C}$, $h_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$, $w_k \sim \mathcal{N}_{\mathbb{C}}(0,\sigma_w^2)$; h_k is assumed known at the DFC and therefore coherent detection is employed. Given these assumptions, $P_{e,k} = \mathcal{Q}(\frac{|h_k|}{2\sigma_w})$ holds. We define the (individual) communication SNR as the (average

[3]This was derived under a high-SNR assumption [7].

[4]We use the term "equivalent" to refer to statistics which are equal up to a scaling factor and an additive term (both independent on $\boldsymbol{y}$ and finite), thus leading to the same performance [11].

individual) received energy divided by the noise power, that is in the i.i.d. case

$$\mathrm{SNR}_k \triangleq \frac{\mathbb{E}\{|h_k b_k|^2\}}{\sigma_w^2} = \frac{P_{D,k}P(\mathcal{H}_1) + P_{F,k}P(\mathcal{H}_0)}{\sigma_w^2}, \quad (15)$$

while in the i.n.i.d. case $\mathrm{SNR}_\star \triangleq \mathbb{E}_{(P_{D,k},P_{F,k})}\{\mathrm{SNR}_k\}$. Here we assume $P(\mathcal{H}_i) = \frac{1}{2}$; the figures are based on 10^6 Monte Carlo runs.

In Fig. 2 we report P_{D_0} vs. P_{F_0} in a scenario with conditionally i.i.d. and i.n.i.d. decisions, respectively[5]. We study a WSN with $K = 10$ and local performance equal to $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ in the i.i.d case while $P_{F,k} \sim \mathcal{U}(0, P_{FU})$, $P_{D,k} = (P_{F,k} + \Delta P)$ and $\Delta P \sim \mathcal{U}(0, P_{DE})$ in the i.n.i.d. case, where $(P_{FU}, P_{DE}) = (0.2, 0.6)$. We report scenarios with $(\mathrm{SNR}_k)_{\mathrm{dB}} \in \{0, 10\}$ (resp. $(\mathrm{SNR}_\star)_{\mathrm{dB}}$, where $\mathrm{SNR}_\star = \frac{P_{FU}+P_{DE}2}{2\sigma_w^2}$ in the i.n.i.d. case). It is apparent that Λ_{LOD} and Λ_{IS} approach Λ_{LRT} at $(\mathrm{SNR}_k)_{\mathrm{dB}} = 0$ in the i.i.d. case (confirming Prop. 1), while there is a moderate loss in the i.n.i.d. case[6]. However, Λ_{IS} suffers from significant loss in performance in both cases $(\mathrm{SNR}_k)_{\mathrm{dB}} = 10$ and $(\mathrm{SNR}_\star)_{\mathrm{dB}} = 10$. Also, in the i.i.d. case Λ_{Wu} is outperformed by both Λ_{CR} and Λ_{LOD}, the latter being the best choice. Finally, the *oscillating behaviour* of Λ_{Wu} is explained since the approximate ML estimate $\hat{P}_D$ (cf. Eq. (9)) is *not reliable* when the WSN is not of large size. Moreover the performance of $\hat{P}_D$ further degrades at low-medium SNR, since $\mathbb{E}\{\hat{P}_D|\mathcal{H}_1\} = \frac{1}{K}\sum_{k=1}^K((1-4P_{e,k}^2)\cdot P_D + 2P_{e,k}^2)$, i.e. when $P_{e,k}^2$ is not negligible, the estimator is *biased* (even if K grows large), as opposed to the exact ML estimate [16].

Fig. 3 shows P_{D_0} vs. $(\mathrm{SNR}_k)_{\mathrm{dB}}$, assuming[7] $P_{F_0} = 0.01$; we simulate a i.i.d. scenario, where $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ and we report the cases $K \in \{10, 30\}$. First, simulations confirm the theoretical findings in Section III: (*i*) only Λ_{IS} and Λ_{LOD} approach Λ_{LRT} at low SNR, while (*ii*) all the considered rules undergo the same performance as the SNR increases. The only exception is given by Λ_{IS}, which keeps close to Λ_{LRT} at low-to-moderate SNR values and exhibits a *unimodal behaviour*, which is consequence of $\lim_{P_{e,k}\to 0}\Lambda_{\mathrm{IS}} = +\infty$, as discussed in Section III. In fact as $P_{e,k} \to 0$, the possible errors are mainly due to the sensing part; on the other hand Λ_{IS} assumes a perfect sensing phase (cf. Eq. (4)), thus misleadingly conjecturing that the whole process is error-free. Finally, Λ_{LOD} is close to Λ_{LRT} over the whole SNR range considered, while Λ_{Wu} has a significant loss in performance and it is always "counter-intuitively" outperformed by Λ_{CR} (with no requirements on system knowledge).

Finally, in Fig. 4 we show P_{D_0} vs. K, assuming $P_{F_0} = 0.01$. We study a i.i.d. setup in the cases $(\mathrm{SNR}_k)_{\mathrm{dB}} \in \{0, 10\}$ (dashed and solid lines, resp.). We analyze the scenarios $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ (scenario A, as in [4]) and $(P_{F,k}, P_{D,k}) = (0.4, 0.6)$ (scenario B, as in [7]). The simulations confirm the performance improvement given by Λ_{LOD} with respect to Λ_{CR} and Λ_{IS} (at the expenses of slightly higher requirements on system knowledge) and the significant improvement with respect to Λ_{Wu} (the latter being *always* outperformed by Λ_{CR}, even when K is large, as proved in Section III). For example, in scenario A with $(\mathrm{SNR}_k)_{\mathrm{dB}} = 0$, Λ_{LOD} achieves $P_{D_0} \approx 0.8$ with $K \approx 30$ sensors as opposed to $K \approx 43$ when Λ_{Wu} is employed.

[5]Note that the concavity of the plots is not apparent, as instead suggested from the theory [11]; this is due to the use of a log-linear scale.

[6]In fact, it can be verified that Prop. 1 does not hold in the latter scenario.

[7]In order to keep a fair comparison, we allow for *rule randomization* whenever its discrete nature does not allow to meet the desired P_{F_0} exactly.

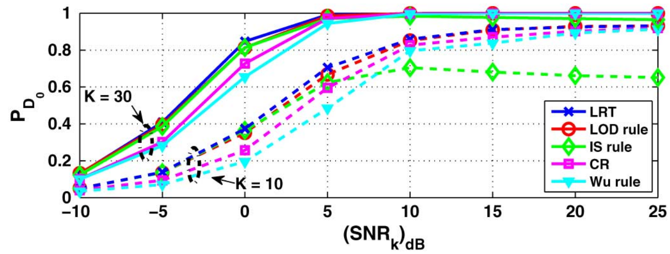

Fig. 3. P_{D_0} vs. $(\mathrm{SNR}_k)_{\mathrm{dB}}$; $P_{F_0} = 0.01$. WSN with $K \in \{10, 30\}$ sensors; $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$.

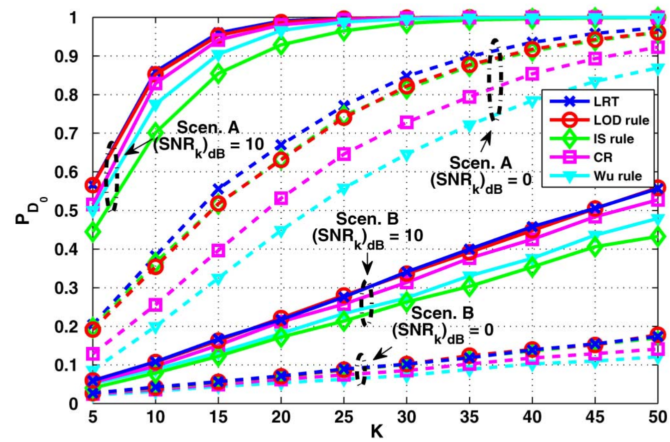

Fig. 4. P_{D_0} vs. K; $P_{F_0} = 0.01$. WSN with $(\mathrm{SNR}_k)_{\mathrm{dB}} \in \{0, 10\}$; $(P_{F,k}, P_{D,k}) = (0.05, 0.5)$ (scen. A) and $(P_{F,k}, P_{D,k}) = (0.4, 0.6)$ (scen. B).

VI. Conclusions

In this letter we studied DF when the DFC knows the false-alarm probability of the generic sensor, but does not the detection probability. Wu rule is always (counter-intuitively, since it makes use of BEPs and false alarm probabilities) outperformed by the simpler counting rule, thus does not exploit effectively the required system parameters. This result is confirmed by a deflection-based analysis, with CR *always* dominating Wu rule, irrespective of the specific BEPs and local performance (in the i.i.d case) considered. Differently, the proposed LOD and IS based rules are appealing in terms of complexity and performance. LOD rule was shown to be close to the clairvoyant LRT over a realistic SNR range (thus *effectively* exploiting knowledge of BEPs and false alarm probabilities), both for conditionally i.i.d. and i.n.i.d. decisions, as opposed to IS rule (only requiring the BEPs for its implementation) being close to the LRT only at low-medium SNR. Optimality of both rules was proved at low SNR in the i.i.d. case, thus motivating the knowledge of false-alarm probability only at medium SNR in a *homogeneous* scenario.

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